Matrix Interpolation based Reduced Order Modelling of a Levitation device with Eddy Current effects

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In this paper, reduced-order models (ROM) based on the proper orthogonal decomposition (POD) are applied to an eddy-current problem with movement. A classical magnetodynamic finite element formulation based on the magnetic vector potential is used as reference and starting point to build up the reduced models. Two approaches are proposed to construct the reduced order models: 1) the so-called classical POD-ROM; and 2) the parametric POD-ROM by matrix interpolation method. The latter is found to be highly computationally efficient (time and memory) when dealing with repetitive computations, such in design, control or optimization procedures. The TEAM workshop problem 28 is chosen as a test case for validation. Results are compared in terms of accuracy and computational cost.

Index Terms—**Parametric reduced-order models, proper orthogonal decomposition, matrix interpolation, eddy currents, electromechanical coupled system, finite-element methods.**

I. Introduction

THE ACCURATE modelling of electromagnetic devices accounting for eddy current effects, non-linearities, movement, is a major concern from early design stages. The finite element (FE) method is widely used and versatile for modelling these phenomena. However, the FE discretization of the problem may result in a large number of unknowns, particularly in three-dimensional problems with different space scales, which is prohibitive to solve in terms of computational time and memory.

Furthermore, the modelling of movement due to electromagnetic forces requires either re-meshing [1], or ad hoc techniques. Without being exhaustive, it is worth mentioning: hybrid finite-element boundary-element (FE-BE) approaches e.g. [2], sliding mesh techniques (with e.g. rotating machines) e.g. [3] or mortar FE approaches e.g. [4].

Reduced-order (RO) techniques with a mathematically-based approach are gaining interest as a feasible alternative in electromagnetic field analysis [5]. Generating the RO model is an expensive task, only justified in case of repetitive analysis, something common in design or optimization procedures where a parametric dependence is crucial. Parametric RO modelling (pROM) is an emerging technique, where the major challenge lays in the fact that, for each new parameter value, we need to generate a new RO model by accessing the full system, thus the computational gain in this case might be inexistent [6]. The aim of the pROM is to accurately approximate the large system by a significantly small one while the parameter dependency in the RO model is preserved [7]. The complete system reduction comprises two stages: offline and online. In the offline stage, the RO models are precomputed for a prescribed set of parameter values from the constructed projection operator or basis; while in the online stage, the solution for a new parameter value is achieved by using the RO models. There are mainly two pROM approaches proposed in the literature [6], [8]. The first approach is the global basis or global projection operator approach, which provides a single basis by concatenating the set of locally computed basis and uses them in the online stage for RO modelling [6]. This approach is not efficient since the RO modelling demands continuous access of the full system for each new parameter values [6]. The second approach is the local basis method where we compute a local basis per prescribed parameter value in the offline stage and we interpolate either the local bases or the RO models constructed from these bases in the online stage for a new parameter value. The local basis or the local RO models cannot be directly interpolated as all of the basis may not correspond to the same coordinates system due to different meshes [8]. Therefore, further re-projection of the basis to a common subspace prior to the interpolation is required. The use of interpolation techniques is very effective in the wider range of the parameter space [9]. The classical interpolation technique can assure the optimal approximation of the linear system, whereas it may fail to preserve non-linear properties [9]. An improved technique for non-linear pROM is based on manifold interpolation which allows to interpolate the subspace corresponding to the basis on a tangent space to a manifold of these subspaces, known as Grassmann manifold interpolation [9], [10].

In [11], authors implement the pROM to design a waveguide filter in the frequency domain. The proposed pROM technique [12] defines the parameter dependent projection matrix within the subdivided parametric space from several bases at predefined interpolation points to use them for generating ROMs with the proper state transformations.

Few RO works have addressed parametric problems in electromagnetic field specially with movement, crucial to model e.g. actuators, rotating electrical machines [13], [14], [15]. A POD-based block-RO model is proposed in [15], [16] to model a moving object and rotating electrical machine respectively,

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where the domain is split in several blocks/sub-domains to construct the small sized corresponding projection bases or projection matrices to apply efficient ROM. In [7] an interesting pROM approach is applied to the dynamic Linear Time Invariant multi-parametric system, where locally reduced order models are generated for several discrete parameter values and re-used. They used matrix matching for dealing with topology change.

In this work, we consider a POD-based FE model of a levitation problem, namely the Team Workshop problem 28 (TWP28) [2], [17], [18] (a conducting plate above two coils, see Fig. 1). In [18] a circuital approach is proposed to reduce the TWP28 problem, where the circuit parameters of self and mutual inductances are evaluated using the FEM and the conductor segmentation method. Two RO models with FE have been proposed. The first RO model deals only with classical POD technique, which has been investigated in our previous work [19]. In this paper, to achieve further computational efficiency, we propose a new POD based single global basis pROM to directly pre-compute the RO models for several prescribed parameter values in an offline stage and then used for the RO modelling by means of a linear matrix interpolation technique in an online stage. The advantage with regard to the technique proposed in [7] is, we only use a single truncated basis to generate the local RO models at several parameter values, therefore, further transformations of the RO models to a local reduced coordinate system are not required. Hence, a simple linear interpolation of the RO models can provide very fast and accurate approximation of the full system in the entire parameter space. Although, we generate subdomain for treating the moving body by adapting the mesh deformation technique and employ RO modelling to the whole domain, in this work the matrix multiplication of the block-MOR [15] with different projection matrices in each time step is completely replaced by the linear interpolation technique. Both classical POD-ROM and pROM models are validated in the time domain and compared in terms of computational efficiency. Although, the proposed approach is presented for a single parameter, the extension to multiple parameters is straightforward.

II. Magnetodynamic levitation model

Let us consider a bounded domain $\Omega = \Omega_c \cup \Omega_c^C \in \mathbb{R}^3$ with boundary Γ. The conducting and non-conducting parts of Ω are denoted by Ω_c and Ω_c^C, respectively. The (modified) magnetic-vector-potential ($a-$) magnetodynamic formulation (weak form of Ampère's law) reads: find a, such that

$$(\nu \operatorname{curl} a, \operatorname{curl} a')_\Omega + (\sigma \partial_t a, a')_{\Omega_c} + \langle \hat{n} \times h, a' \rangle_\Gamma = (j_s, a')_{\Omega_s}, \ \ \forall a' \quad (1)$$

with a' test functions in a suitable function space; $b(t) = \operatorname{curl} a(t)$, the magnetic flux density; $j_s(t)$ a prescribed current density imposed in a source domain Ω_s and $\hat{n}$ the outward unit normal vector on Γ. Volume integrals in Ω and surface integrals on Γ of the scalar product of their arguments are denoted by $(\cdot,\cdot)_\Omega$ and $\langle \cdot,\cdot \rangle_\Omega$. The derivative with respect to time is denoted by ∂_t. We further assume linear isotropic and time independent materials with magnetic constitutive law $h(t) = \nu b(t)$ (magnetic field h, reluctivity ν) and electric constitutive law $j(t) = \sigma e(t)$ (electric field $e(t) = -\partial_t a$, conductivity σ). The unknown $a(t)$ is discretized with edge elements (in the general 3D case) and with nodal elements (in the 2D case). A Galerkin approach is adopted. Assuming a rigid body Ω_c (no deformation) and a purely translational movement (no rotation, no tilting), the electromagnetic force appearing due to the eddy currents in Ω_c can be modelled as a global quantity with only one component (perpendicular to the plate). In this work, the movement is treated with Lagrangian approach. If Ω_c is non-magnetic, Lorentz force can be used:

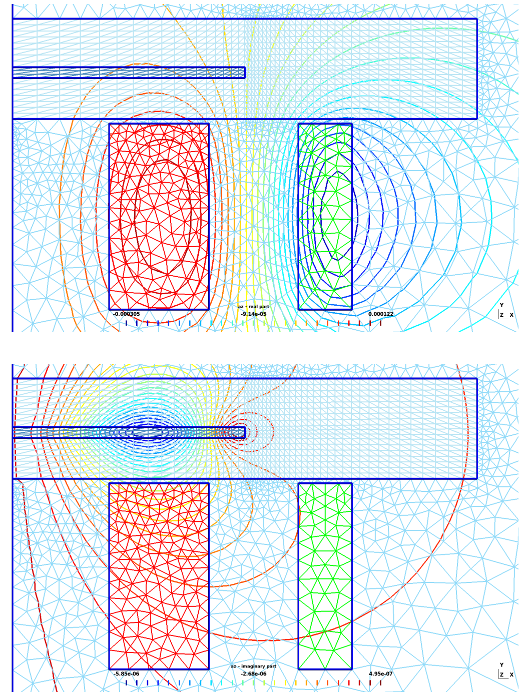

Fig. 1. 2D axisymmetric mesh of TWP28: aluminium plate above two concentric coils (12.8 mm clearance). Real part (up) and imaginary part (down) of the magnetic flux density (detail of the deformed mesh subdomain).

$$F_{em}(t) = \int_{\Omega_c} j(t) \times b(t) \, \mathrm{d}\Omega_c = \int_{\Omega_c} -\sigma \partial_t a \times \operatorname{curl} a \, \mathrm{d}\Omega_c \, . \quad (2)$$

The 1D mechanical equation governing the above described levitation problem reads:

$$m \, \partial_t v(t) + \xi v(t) + \kappa y(t) + mg = F_{em}(t) \quad (3)$$

where the moving body (with mass m) is at unknown position $y(t)$, moving with speed $v(t) = \partial_t y(t)$, g is the gravitational acceleration, ξ is the scalar viscous friction coefficient, κ is the elastic constant. We apply the backward Euler method to solve (3). The solution of (1), $a(t)$, is used to compute the electromagnetic force (2) that gives rise to the movement and the position $y(t)$ in (3) can be computed. Given that, the dynamics of the mechanical equation (3) is much slower than the dynamics of the electromagnetic equation (1), if the time-step is taken sufficiently small, one can decouple both equations and solve them alternatively rather than simultaneously by the weak electromechanical coupling algorithm of [20]. We adopt this approach.

III. POD-BASED MODEL ORDER REDUCTION

The proper orthogonal decomposition (POD) is applied to reduce the matrix system resulting from the FE discretisation of (1):

$$A\partial_t x(t) + Bx(t) = c(t)\,. \tag{4}$$

where $x(t) \in \mathbb{R}^{N\times 1}$ is the time-dependent column vector of N unknowns (discretized magnetic vector potential $a(t)$), A, $B \in \mathbb{R}^{N\times N}$ are symmetric positive semidefinite and symmetric positive definite matrices respectively and $c(t) \in \mathbb{R}^{N\times 1}$ is the source column vector. Furthermore, the system (4) is discretized in time by means of the backward Euler scheme. A system of algebraic equation is obtained for each time step t_k, $\Delta t = t_k - t_{k-1}$ the step size. The time discretized system reads:

$$\left[A_{\Delta t} + B\right] x_k = A_{\Delta t} x_{k-1} + c_k\,, \tag{5}$$

with $A_{\Delta t} = \frac{A}{\Delta t}$, $x_k = x(t_k)$ the solution at instant t_k, $x_{k-1} = x(t_{k-1})$ the solution at instant t_{k-1}, $c_k = c(t_k)$ the right-hand side at instant t_k.

In RO techniques, the solution vector x_k is approximated by a vector $x_k^R \in \mathbb{R}^{V\times 1}$ within a reduced subspace spanned by $\Psi \in \mathbb{R}^{N\times V}$, $V \ll N$,

$$x_k \approx \Psi x_k^R\,. \tag{6}$$

with Ψ an orthonormal projection operator generated from the time-domain full solution $x(t)$ via the snapshots technique [21].

Let us consider the simple case, where the spatial discretization (mesh) for all time steps is the same (no movement), so the snapshot matrix is, $S = [x_1, x_2, \ldots, x_V] \in \mathbb{R}^{N\times V}$ from the set of solutions $x_k \in \mathbb{R}^{N\times 1}$ for the selected number of time steps, where $k \in [1\cdots V]$. Applying the singular value decomposition (SVD) to S as,

$$S = \mathcal{U}\Sigma\mathcal{V}^T\,. \tag{7}$$

where $\Sigma \in \mathbb{R}^{N\times V}$ is a diagonal matrix which contains the singular values, ordered as $\lambda_1 > \lambda_2 > \ldots > 0$, $\mathcal{U} \in \mathbb{R}^{N\times N}$ and $\mathcal{V} \in \mathbb{R}^{V\times V}$ are left and right orthonormal matrices respectively. We consider $\Psi = \mathcal{U}^r \in \mathbb{R}^{N\times M}$, that corresponds to the truncation of $\mathcal{U}$ (i.e. its M first columns, which has larger singular values than a pre-defined error tolerance ε) Therefore, the RO system of (5) reads

$$\left[A^r_{\Delta t} + B^r\right] x^r_k = A^r_{\Delta t} x^r_{k-1} + c^r_k\,, \tag{8}$$

where

$$A^r_{\Delta t} = \Psi^T A_{\Delta t}\Psi,\; B^r = \Psi^T B\Psi,\; c^r = \Psi^T c \tag{9}$$

[22].

Note that the rank of $x_k^r \in \mathbb{R}^{M\times 1}$ in (8) is much smaller than the one of $x_k^R \in \mathbb{R}^{V\times 1}$ in (6), $M \ll V$.

RO model of an electromagnetic problem with movement

The POD-RO modelling of an eddy-current problem with movement must handle the changing geometry/mesh per time step. Indeed, the supporting mesh depends on the position of the moving part, matrices in (4) vary along y. To treat this issue the mesh can either be 1) fixed but non-conforming (mortar method); or 2) deformed in limited domain.

1) Classical POD-RO modelling

The POD-RO modelling starts with the selection of solutions of the full time domain FE problem (from transient to steady state), so-called snapshots. In case of movement modelled with re-meshing, the number of unknowns per time step may vary and the construction of the snapshot matrix S requires an intermediate time step. As the solutions at t_k are linked to different meshes, i.e. one mesh per instant t_k. the snapshot vectors x_k have a different size. Hence, to assemble the snapshot vectors in S in order to generate projection operator Ψ, the vectors have to be projected to a common basis. Although a simple linear interpolation technique can solve the rank problem [19], the RO model construction still needs Ψ to be projected to a common basis at each time step. Therefore, the procedure becomes completely inefficient on computational basis.

To overcome this issue, we adopt a mesh deformation technique (constraint re-mesh), limited to a region around the moving body (see, e.g., the box in Fig. 2). Therefore, the moving body moves as a block with its mesh and the elements around are deformed. If the body moves upward, elements above the plate are shrinked, elements under the plate are expanded and the other way around. The re-meshing is done by deforming a high quality initial mesh, which is generated with the conducting plate placed at, e.g., y_0, center of the re-meshing region (avoiding bad quality elements), see Fig. 2. The surrounding mesh does not vary. Further details can be found in [19].

2) POD-RO modelling with Matrix Interpolation

The parameter dependent solution changes with each parameter value, therefore, parametric reduced order modelling (pROM) becomes more challenging due to the expensive basis generation stage. However, once the pROM is available it is then much more computationally efficient (in terms of time) to solve the system for any parameter values. Such approach is interesting when dealing with repetitive computations, such in design or optimization procedures.

Regarding non-parametric POD-RO modelling, it is sufficient to construct a single Ψ from the best chosen snapshot vectors in order to pre-compute the constant reduced matrices (9) only once. In parametric POD-RO case, on the contrary, pre-computation of the reduced matrices is not possible as, e.g., the matrices A, B in (4) change with the new parameter values, e.g., the position y. Hence, the matrices multiplication (9) are held for each new y, which makes the RO modelling extremely expensive. Although, a considerable computational gain can be achieved on RO modelling over the full system solution if the size of Ψ is much lower than the full system size.

In order to cope with the second difficulty of efficient pROM, i.e., parametric dependence of electromagnetic field on moving body position y, we propose a new approach where in the offline stage: a single global basis is generated by POD technique from S snapshot matrix which can be directly used to construct several RO models for a chosen number of parameter values; in online stage: these RO models are interpolated for the entire parametric space. The proposed method is presented in algorithmic steps in algorithm 2.

Note that the RO model evaluation in the time-stepping schemes needs $A^r_{\Delta t}$, c^r and B^r. The classical POD-ROM

Algorithm 1: Classical POD-ROM

In : snapshot matrix $S = [x_1, \dots, x_V] \in \mathbb{R}^{N \times V}$,
$x_k \in \mathbb{R}^{N \times 1}$
time steps $\{t_k\}$, $k \in [1, ..., K]$
$A_{\Delta t},\ c$,
tolerance ε
$V \leq N$ snapshot vectors
Out: displacement y_k

1 $y_0 =$ initial position, $\Delta y_0 = 0$
2 get initial mesh
3 SVD of $S = \mathcal{U}\Sigma\mathcal{V}^T$
4 $\Psi = \mathcal{U}(:, 1 \dots M)$ with M such that $\lambda(i)/\lambda(1) > \varepsilon, \forall i \in [1 \dots M]$
5 $A^r_{\Delta t} = \Psi^T A_{\Delta t} \Psi,\ c^r = \Psi^T c$
//Time resolution
6 **for** $k \leftarrow 1$ **to** K **do**
//Magnetics
7 generate matrices B_k
8 $B^r_k = \Psi^T B_k \Psi$
9 solve $\left(A^r_{\Delta t_k} + B^r_k\right) x^r_k = c^r + A^r_{\Delta t_k} x^r_{k-1}$
10 $x_k \approx \Psi x^r_k$
11 compute force F_k
//Mechanics
12 compute displacement y_k
13 update $\Delta y_k = y_k - y_{k-1}$
14 deform mesh with y_k
15 **end**

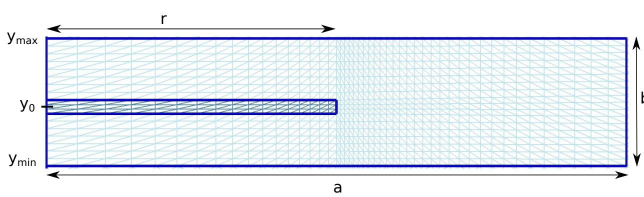

Fig. 2. Sub-domain for deformation: plate position at $y_0 = 12.8\,\text{mm}$ (initial structured mesh).

method (Algorithm 1) generates matrices B^r_k for each time step k (lines 7,8), while the pROM approach (Algorithm 2) interpolates B^r_k (lines 11,12). Therefore, Algorithm 2 avoids the expensive full order matrix multiplication by means of the linear interpolation technique. Both matrices (A, B) depend on the space discretization, and therefore on the mesh. The difference is that A has time derivative but no space derivative and B has space derivate (curl-curl) but no time derivative. Apart from that, the material of the plate is non-magnetic, so the reluctivity of the plate and the air is the same so that A is independent of the position, invariant in space.

IV. Application Example

We consider TWP28: an electrodynamic levitation device consisting of a conducting cylindrical aluminium plate ($\sigma = 3.47 \cdot 10^7$ S/m, $m = 0.107\,$Kg, $g = 9.81\,\text{m/s}^2$, $\xi = 1$ N·m·s, $\kappa = 0$) above two coaxial exciting coils. The inner and

Algorithm 2: pROM

In : snapshot matrix $S = [x_1, \dots, x_V] \in \mathbb{R}^{N \times V}$,
$x_k \in \mathbb{R}^{N \times 1}$
time steps $\{t_k\}$, $k \in [1, ..., K]$
$A_{\Delta t},\ c$,
positions y_l, $l \in [1, ..., p]$, $\Delta y = \dfrac{|y_{max} - y_{min}|}{p}$
$\{B(y_l)\}$, $B(y_l) \in \mathbb{R}^{N \times N}$
tolerance ε
$V \leq N$ snapshot vectors
Out: displacement y_k

1 $y_0 =$ initial position, $\Delta y_0 = 0$
2 get initial mesh
3 SVD of $S = \mathcal{U}\Sigma\mathcal{V}^T$
4 $\Psi = \mathcal{U}(:, 1 \dots M)$ with M such that $\lambda(i)/\lambda(1) > \varepsilon, \forall i \in [1 \dots M]$
5 $A^r_{\Delta t} = \Psi^T A_{\Delta t} \Psi,\ c^r = \Psi^T c$
6 **for** $l \leftarrow 1$ **to** p **do**
7 $\{B^r(y_l)\} \leftarrow \Psi^T B(y_l) \Psi$, with, $B^r(y_l) \in \mathbb{R}^{M \times M}$
8 **end**
//Time resolution
9 **for** $k \leftarrow 1$ **to** K **do**
//Magnetics
10 find $y_i = \min(|y_l - y_{k-1}|), \forall l$
//Linear Interpolation
11 i) If $y_i > y_{k-1}$ then
$B^r_k = B^r(y_{i-1})(1-\theta) + B^r(y_i)\theta$, with,
$\theta = 1 - \dfrac{|y_{k-1} - y_i|}{\Delta y}$
ii) If $y_i < y_{k-1}$ then
$B^r_k = B^r(y_i)(1-\theta) + B^r(y_{i+1})\theta$, with,
$\theta = 1 - \dfrac{|y_{k-1} - y_i|}{\Delta y}$
iii) If $y_i == y_{k-1}$ then $no\ interpolation$
12 solve $\left(A^r_{\Delta t} + B^r_k\right) x^r_k = c^r + A^r_{\Delta t} x^r_{k-1}$
13 $x_k \approx \Psi x^r_k$
14 compute force F_k
//Mechanics
15 compute displacement y_k
16 update $\Delta y_k = y_k - y_{k-1}$
17 deform mesh with y_k
18 **end**

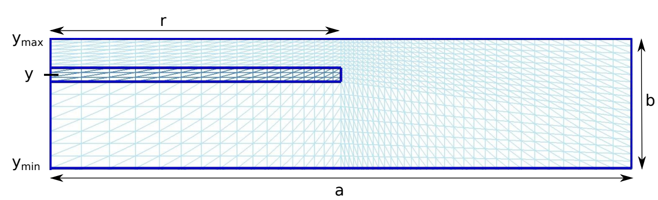

Fig. 3. Sub-domain for deformation: plate position at $y = 20\,\text{mm}$. Structured mesh elements under the plate are expanded and above the plate are shrinked.

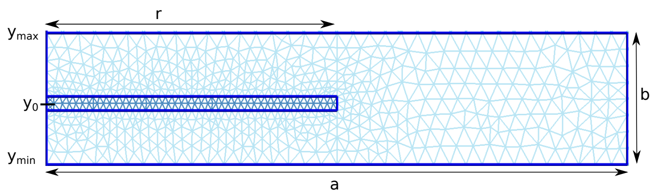

Fig. 4. Sub-domain for deformation: plate position at $y = 12.8\,\mathrm{mm}$ (initial unstructured mesh).

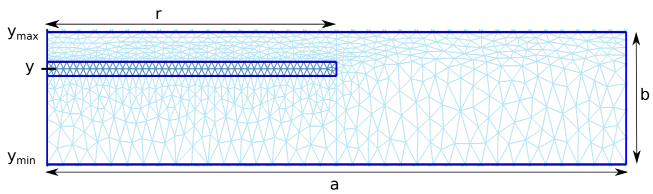

Fig. 5. Sub-domain for deformation: plate position at $y = 20\,\mathrm{mm}$. Unstructured mesh elements under the plate are expanded and above the plate are shrinked.

outer coils have 960 and 576 turns respectively. At $t = 0$, the plate rests above the coils at a distance of 3.8 mm. For $t \geq 0$, a time-varying sinusoidal current (20 A, $f = 50\,\mathrm{Hz}$) is imposed, same amplitude, opposite directions [17]. Assuming a translational movement (no rotation and tilting) we can use an axisymmetric model. A FE model is generated as reference and origin of the RO models. We have time-stepped 50 periods (100 time steps per period and step size 0.2 ms), discretization that ensures accuracy and avoids degenerated mesh elements during deformation. Note that, the application of the method is general, valid for a full three-dimensional problem.

A. Mesh deformation of a sub-domain for POD-RO model

The choice of the sub-domain to deform the mesh is a non-trivial task: it should be as small as possible while ensuring a high accuracy. From our reference FE solution [17] available measurements, or using an approximate analytical solution, e.g. a circuital approach an equivalent circuit for getting an approximated solution, by observing the minimum and maximum levitation height of the plate, we fixed the sub-domain size along the $y-$axis between $y_{min} = 1.3\,\mathrm{mm}$ and $y_{max} = 29.3\,\mathrm{mm}$, distances measured from the upper border of the coils. The size along the $x-$axis has a minimum equal to the radius of the plate, i.e. $r = 65\,\mathrm{mm}$. This value is however not enough due to fringing effects. Trying to observe the influence of the sub-domain size for RO models, we have taken different sizes along the $x-$axis: $1.5r$, $2r$, $3r$ (97.5, 130, 195 mm), measured from the axis. The box is meshed for these three values with 1667, 1566, 1509 number of nodes, yields 1921, 1836 and 1780 unknowns, respectively. Considering the first peak (1P) of the time step solutions in to the snapshot matrix, i.e. $S \in 1\mathrm{P}$ the projection operator Ψ is generated, three RO models for three sub-domain lengths are then evaluated and compared. We can clearly observe from Table I that the larger the sub-domain (box) size the smaller the relative error is. We have therefore chosen to further analyse the RO results obtained with a box length along x of 195 mm ($3r$).

TABLE I
L_2-RELATIVE ERRORS OF RO MODELS ON LEVITATION HEIGHT FOR 1P (MESH DEFORM).

sub-domain lengths (mm)	$M = 7$	$M = 35$
97.5	$8.24 \cdot 10^{-2}$	$6.14 \cdot 10^{-4}$
130	$5.71 \cdot 10^{-2}$	$1.90 \cdot 10^{-4}$
195	$4.53 \cdot 10^{-3}$	$3.73 \cdot 10^{-5}$

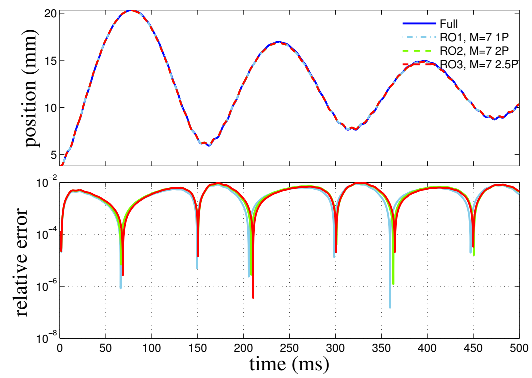

Fig. 6. Displacement (up) and relative error (down) between full and RO models with structured mesh (M=7).

The number of unknowns remain same for all RO models computation.

The accuracy of the RO system mostly depends on the selection of the most important dynamic information of the full system to the snapshot matrix. An automatic and cost efficient snapshot selection is possible by using a greedy algorithm developed in [22] in case of a reasonable number of training points (e.g: time steps). Nevertheless, using a greedy algorithm for the system with long transient behaviour would be extremely expensive as the greedy procedure solves the RO model for V times at every time step to construct the basis vectors of size V. In our test case we have similar kind of situation where the plate reaches at steady state after several periods of damping oscillation. Therefore, from a cheap magnetic equivalent circuit test setup we can have an idea about the time step solutions which includes the first transient peak of the full system. Initially the first 8 number of time periods (160 ms) of the simulation, that correspond to the first damping oscillatory peak (1P) are included to S. However, further transient peaks (2P and 2.5P) inclusion to S does not improve the accuracy of RO models, but the accuracy significantly improves with M, see in Fig 6 and 7. The basis are truncated as $\Psi = \mathcal{U}^r$ (M first columns) by means of prescribed error tolerance ($\varepsilon = 10^{-5}$) for $M = 7$ and tolerance ($\varepsilon = 10^{-8}$) for $M = 35$. It can be observed from Fig 6 that only $M = 7$ basis from $S \in 1\mathrm{P}$ can approximate the full system accurately, whereas adding more number of basis, e.g., $M = 35$ leads the relative error to the range of 10^{-5}. The mesh deformation technique can also be implemented to an unstructured mesh (see Fig 4) to approximate the full system behaviour accurately with the basis $M = 35$ in Fig 8.

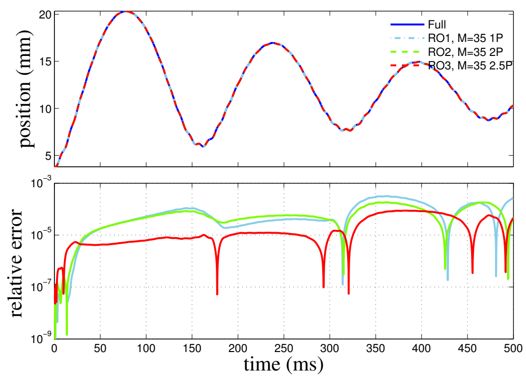

Fig. 7. Displacement (up) and relative error (down) between full and RO models with structured mesh (M=35).

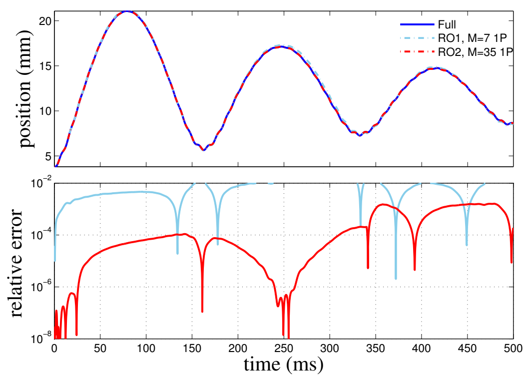

Fig. 8. Displacement (up) and relative error (down) between full and RO models with unstructured mesh for 1P

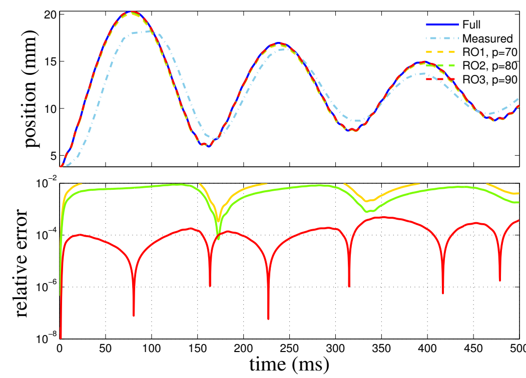

Fig. 9. Displacement (up) and relative error (down) between full, measured and RO models using matrix interpolation with structured mesh for 1P and $p = 70, 80, 90$ number of positions with $M = 35$ basis.

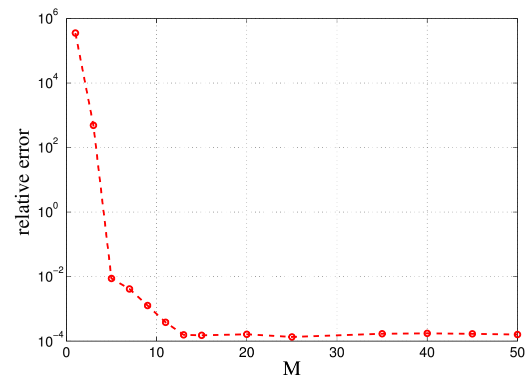

Fig. 10. RO models convergence rate with 1P (pROM).

B. Linear interpolation of coefficient matrices for POD-RO model

We already have observed that, in order to select the optimum snapshot vectors for generating Ψ, we have to include the first peak (1P) of the time step solutions in to the snapshot matrix. Then the basis is truncated by means of prescribed error tolerance ($\varepsilon = 10^{-8}$) to get RO models of size $M = 35$.

In the matrix interpolation RO approach, the position dependent matrices, i.e. $B(y)$ in our test case, are precomputed for $p = 70, 80, 90$ (with $\Delta y = 0.0003$, 0.00025, 0.0002 respectively) equally distributed positions in between the minimum and maximum levitation height, which are then reduced. The RO models are interpolated to achieve the system behaviour accurately, see in Fig 9 for entire parametric space. It can be observed from Fig 9 that the more we increase the number of positions p, the more accurate the results will be. All simulations have been performed on a laptop, Intel Core i7-4600U CPU at 2.10 GHz, without parallelization. The computational time and cost can be significantly reduced by substituting the system matrices multiplication with the linear interpolation technique. RO models convergence rate and the computational gain can be observed in Fig 10 and 11 respectively. The basis generation time for the classical POD-ROM is 32.1 min with 10.47 MB storage space and the basis and RO models generation time for pROM (matrix interp.) is 33.8 min with 19.18 MB storage space. The total computational time of ROM and pROM for $M = 35$ is 101.1 min and 55.84 min respectively. Note that, the ROM time comprises of the offline stage (basis generation) and the online stage (ROM simulation time for the entire parameter space). Therefore, the basis generation for pROM is expensive compare to the ROM approach, however, once the pROM is achieved the RO modelling becomes highly efficient in terms of the computational time. The TWP28 problem reaches steady state after 1600 ms [17]. Herein we have shown only the first 500 ms (2500 time steps), that took 125 min computational time without model order reduction technique, 68.95 min ($M = 35$) with classical POD-ROM and 22.69 min ($M = 35$) with pROM, see in Fig 11. If we increase the number of time steps, the gain achieved will be higher, as the basis generation time remains same.

V. Conclusion

In this paper, we have proposed two approaches: classical POD-ROM and pROM for POD-based RO models to treat a magnetodynamic levitation problem with deformation of a subdomain around a moving body technique. The approach with classical POD-ROM has proved to be accurate and efficient (low computational cost in terms of time and storage compare to the full system), as the full system can be projected in to a

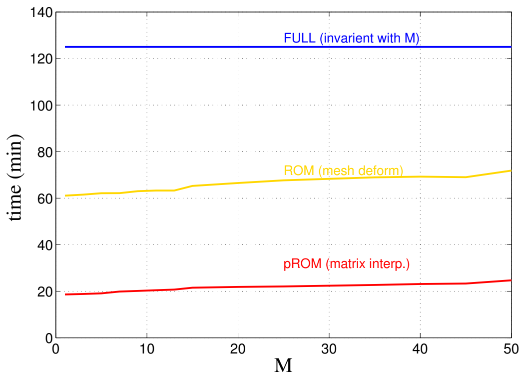

Fig. 11. Computational time (2500 time steps) with respect to the reduced basis size.

very small subspace. We have shown results for three different sub-domain sizes on which the bigger the sub-domain is the higher the accuracy is achieved. Further, accurate and computationally efficient (with regard to time once the reduced model is available) RO model has been developed by means of pROM with matrix interpolation approach, which is 6.3 times faster than the full system solution. It has been observed that the basis generation in offline stage requires considerable time and storage for both classical POD-ROM and pROM approaches, therefore, the actual computation cost for RO modelling is still high. The computational cost can be dramatically reduced by reusing of generated RO models for repetitive computation of the system in online stage.

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